AUGMENTATION OF HIGH-ALTITUDE MANEUVER PERFORMANCE OF A TAIL-CONTROLLED MISSILE USING LATERAL THRUST

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Tactical ballistic missiles (TBM's) may experience severe spiral maneuvers as they reenter the earth's atmosphere. It has been estimated that these maneuvers may increase in magnitude from 1 to 10 g's as the vehicle descends in altitude from 100 to 60 kft. The maneuvers also occur within the frequency range 0.5 to 1.0 Hz. This imposes the greatest difficulty on present-day proportional navigation interceptors. To hit these targets the interceptor must possess extremely fast maneuver response characteristics. At the altitudes mentioned this is generally impossible with aerodynamic control. This study investigates the possibility of achieving the desired interceptor time constant with a combination of aerodynamic and lateral-thrust control. The basic concept involves the use of conventional aerodynamic control for most of the terminal engagement with transition to blended control as time-to-go approaches zero. As a modification to existing airframes, the feasibility of the concept depends on the peak level of the thrust required, the location of the thrust device along the missile body, and the necessary total impulse. These factors are addressed in relation to a typical tail-controlled interceptor. A simple blended-autopilot control theory is developed and the improvement in miss distance performance that results from the concept is indicated. Two of the more important findings of the study are that there is an optimum location along the missile body for the point of application of the thrust and that the time-to-go for transition from aerodynamic to blended control should approximate 0.5 sec.

INTRODUCTION

The maneuver profile of the reentry tactical ballistic missile (TBM) is of prime importance in establishing interceptor design requirements. Some indication of the problem is provided by Figure 1 which shows the Mach number and maneuver stiffness of a typical TBM during the last 150,000 ft of reentry.
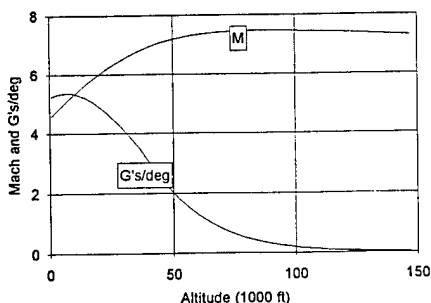

FIGURE 1. MACH NUMBER AND MANEUVER STIFFNESS. MEDIUM-RANGE TBM

The spiral dynamics of nonseparating TBM's as they reenter the atmosphere have been described in detail elsewhere[1]. The problem is one of small static stability at low angles of attack. Indeed, at high reentry Mach, some of these vehicles are believed to possess a region of negative static stability at incidences less than about 6 deg. In the absence of a strong aerodynamic restoring moment, small accidental airframe asymmetries may produce large rolling trim amplitudes and, as indicated above, high spiral g levels. At altitudes above 100,000 ft however the effects of airframe asymmetry are relatively unimportant and the lateral motion of the missile is characterized almost entirely by the rapidly diminishing exoatmospheric transient. At 100,000 ft a residual plane transient oscillatory motion of frequency 0.25 to 0.5 Hz and amplitude 5 to 10 deg. may be assumed[1] From Figure 1, a lateral maneuver level of about 1-g may thus be expected.

Below 80,000 ft the transient has disappeared and the missiles lateral motion depends mainly on its static stability characteristics and the degree of configurational asymmetry in the airframe. The perfectly symmetric missile with positive static stability may thus be assumed to align precisely with the velocity vector. In the presence of configurational asymmetries, however, the missile will

experience a circular pitching and yawing motion at the roll frequency. With positive static stability, the amplitude of this lunar motion depends critically on the degree of asymmetry and the ratio of accidental roll rate to natural pitch frequency. If the statically stable missile exhibits the phenomenon of roll lock-in at the critical frequency, sustained circular yaw of amplitude at least 8 deg seems possible. In the event the missile is statically unstable at yaw levels less than about 6 deg, yaw amplitudes of at least 6 to 8 deg will occur irrespective of the degree of asymmetry.

With this brief summary of the results of Reference 1, it is suggested that the upper endoatmospheric TBM interceptor be capable of engaging 1,3 and 8-g spiraling targets at altitudes of 100,000, 80,000 and 60,000 ft, respectively. The frequency of the target maneuvers should be considered to occur in the range 0.5 to 1 Hz. Engaging these targets with less than 1 ft of miss presents an impossible task for present-day aerodynamically controlled interceptors. This limitation is of considerable importance and may be demonstrated with the following simple example:

Consider a typical tail-controlled proportional navigation interceptor with the general inertia and aerodynamic properties listed in Table 1.

TABLE 1. INTERCEPTOR PARAMETERS

m	I	$C_{M\delta}$	$C_{M\alpha}$	$C_{Z\delta}$	$C_{Z\alpha}$	δ_{MAX}	α_{MAX}
25	550	-8	-12.5	-1.5	-25	45	30

At 80,000 ft altitude at 4000 ft/sec the dynamic pressure is 700 lb/ft^2 and the pitch control moment derivative is $M_\delta =$ -5600 ft lb/rad. The lift curve slope is 17,500 lb/rad and the static restoring moment derivative is $M\alpha=-8750$ ft lb/rad. The trim angle of attack (AOA) with the tails hard over is $\alpha_T=30$ deg and the corresponding steady-state lateral acceleration 10.5 g's. The equation for pure pitch is simply

$$I\ddot{\alpha} = M_\delta \delta + M_\alpha \alpha \qquad (1)$$

and if we assume the tail drives at 400 deg/sec until δ reaches 45 deg, there may be computed $\tau=0.36$ sec as the time required for α to reach 66 percent of α_T. Assume this to be the equivalent of a single-lag time constant and

substitute in the equation for the noise-free miss of the infinitely maneuverable single lag proportional navigation interceptor

$$M = \frac{A\omega\tau^3}{(1+\omega^2\tau^2)^{3/2}} \qquad (2)$$

where A is the target acceleration amplitude, ω_O is the weave frequency and τ is the interceptor single-lag time constant. Against a 3-g weave at 0.5 Hz (previously specified target at 80,000 ft altitude) with $\tau=0.36$ sec, the miss is 5 ft or about one-half the lateral excursion amplitude of the target. With more realistic tail-control dynamics the actual miss will be considerably larger, as shown later. In our example, the interceptor steady-state maneuver capability was noted to be 10.5 g's, which is well within the range usually considered necessary to engage a 3-g target[2]. The problem is not one of insufficient g-capability. It is one of agility or speed of response. Consequently, the main reason for the proposed lateral-thrust concept is that an extremely rapid acceleration response may be achieved at high altitudes where the missile's aerodynamic control effectiveness is weak. The present design stratagem is to employ this lateral thrust to achieve a rapid initial response and then to rely on the tails to maintain the desired acceleration command in the steady state. The tails are also used to maintain dynamic stability. A theoretical account is given of the dynamics of a tail-controlled missile employing blended aerodynamic and lateral thrust. For this purpose, a second-degree autopilot is assumed. The interceptor speed is 4000 ft/sec. The gains are determined at each altitude to provide the fastest step response without violating specified constraints on tail deflection and peak thrust level. For the interceptor of the present study, an attempt is made to establish both the desired magnitude of the thrust required and to locate the best position along the missile body for the point of application of the thrust vector. An indication is given of the improvement in miss distance performance that may be expected with blended control against the specified weaving targets.

BLENDED CONTROL

In this analysis proportional lateral thrust P at distance b ahead of the missile center of gravity is assumed, as shown in Figure 2. We also assume linear

aerodynamics and neglect jet interference effects. The control law is

$$P = K(A - A_c) \tag{3}$$

$$\delta = k_1(A - \varepsilon A_c) + k_2\dot{\alpha} \tag{4}$$

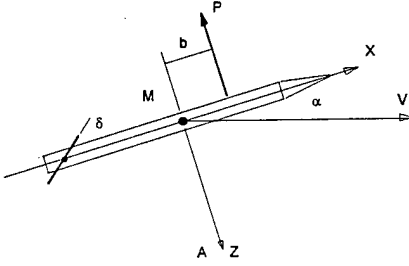where A_C is the command acceleration.

FIGURE 2. PRIMARY VARIABLES

For convenience, we neglect the effect of plunging on the angular motion of the missile ($\dot{\theta} = \dot{\alpha}$) and write the equation of angular momentum in the form

$$I\ddot{\alpha} = M_\alpha\alpha + M_\delta\delta + bP \tag{5}$$

The total lateral acceleration A is

$$mA = Z_\alpha\alpha + Z_\delta\delta - P \tag{6}$$

Define the following non-dimensional variables

$$m_\alpha = \frac{M_\alpha}{mk^2} \quad m_\delta = \frac{M_\delta}{mk^2} \quad z_\alpha = \frac{Z_\alpha}{m}$$

$$z_\delta = \frac{Z_\delta}{m} \quad r = \frac{b}{k^2} \quad p = \frac{P}{m} \tag{7}$$

where k is the transverse radius of gyration. Then Equations 3 to 6 become

$$mp = K(A - A_c) \tag{8}$$

$$\delta = k_1(A - \varepsilon A_c) + k_2\dot{\alpha} \tag{9}$$

$$\frac{d^2\alpha}{dt^2} = m_\alpha\alpha + m_\delta\delta + rp \tag{10}$$

$$A = z_\alpha\alpha + z_\delta\delta - p \tag{11}$$

In the steady state at constant *AOA* ($A=A_C$ and $p=0$) there results immediately

$$\varepsilon = 1 - \frac{m_\alpha}{k_1(m_\alpha z_\delta - m_\delta z_\alpha)} \tag{12}$$

Writing the homogeneous differential equation as

$$\frac{d^2A}{dt^2} + B_1\frac{dA}{dt} + B_2A = 0 \tag{13}$$

the coefficients are

$$B_1 = -k_2\frac{(1+K)m_\delta + rK z_\delta}{(1+K) - k_1 z_\delta} \tag{14}$$

$$B_2 = \frac{k_1(m_\alpha z_\delta - m_\delta z_\alpha) - rK z_\alpha - m_\alpha(1+K)}{(1+K - k_1 z_\delta)} \tag{15}$$

If the previous differential equation is written in the form

$$\frac{d^2A}{dt^2} + 2\xi\omega_o\frac{dA}{dt} + \omega_o^2 A = 0 \tag{16}$$

where ξ and ω_O are the usual damping and frequency concepts, the control gains k_1 and k2 may be derived as

$$k_1 = \frac{\omega_o^2(1+K) + rK z_\alpha + m_\alpha(1+K)}{(m_\alpha z_\delta - m_\delta z_\alpha) + \omega_o^2 z_\delta} \tag{17}$$

$$k_2 = \frac{-2\xi\omega_o(1+K - k_1 z_\delta)}{rK z_\delta + m_\delta(1+K)} \tag{18}$$

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RESPONSE TO UNIT STEP COMMAND

The missile's lateral acceleration step response has been optimized assuming a constant speed of 4000 ft/sec at the three altitudes mentioned earlier, namely 100,000, 80,000 and 60,000 ft. The missile's mass parameters and aerodynamic stability coefficient derivatives were given in Table 1. Maximum acceptable tail deflection and lateral control thrust are 45 deg and 4000 lb, respectively. The corresponding trim AOA with the stability derivatives of Table 1 is 30 deg. Aerodynamic trim at 100,000 ft altitude at 4000 ft/sec is 4.2 g's; at 80,000 ft, it is 10.5 g's; at 60,000 ft, it is 26 g's. These maneuver capabilities are generally well within the levels considered necessary to engage the targets mentioned. The problem is not one of insufficient g-capability; it is one of agility or speed of response.

At 100,000 ft or the maximum altitude considered the dynamic pressure is 285 lb/sq ft. For this flight condition the values of the closed-loop undamped natural frequency ω_O and control thrust gain K, which yield the fastest time response without violating the specified constraints, are 5 rad/sec and 1.5, respectively. A description of how these autopilot parameters were determined is given later. With these parameters, Figure 3 (left) shows the missile's blended response to a 4.2-g step command At the right in the figure, similar results are shown for aerodynamic control using tails only with $K=0$ and $\omega_O=3$ rad/sec. (While not included in the previous analysis, the results of Figure 3 include the effect of a first-order time lag of 20 msec on both the thrust and tail deflection). The time constant with tail control is about 0.7 sec. Increasing the order of the autopilot from the second degree to the third degree or higher will not change this result significantly. The time constant is determined more by the basic aerodynamic and inertia properties of the airframe than by the detailed characteristics of the autopilot. By combining tail control with lateral thrust the time constant is reduced to 0.1 sec. Note that at this extremely low dynamic pressure peak thrust is much less than the 4000 lb assumed to be available. The interesting manner in which the lateral acceleration develops, first through an almost step increase due to thrust and then increasingly from AOA is shown in Figure 4.

SELECTION OF THE AUTOPILOT PARAMETERS

The general procedure for establishing K and ω_O is illustrated in Figure 5. These results are for the flight condition H=80,000 ft and V=4000 ft/sec.
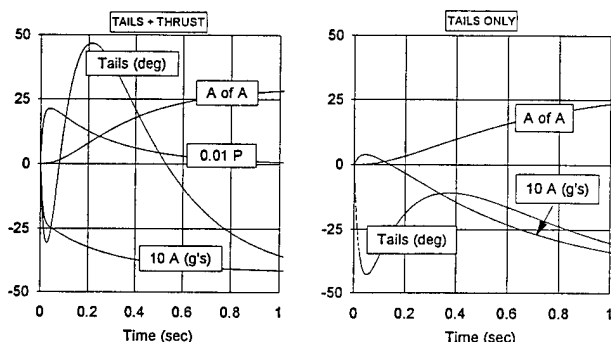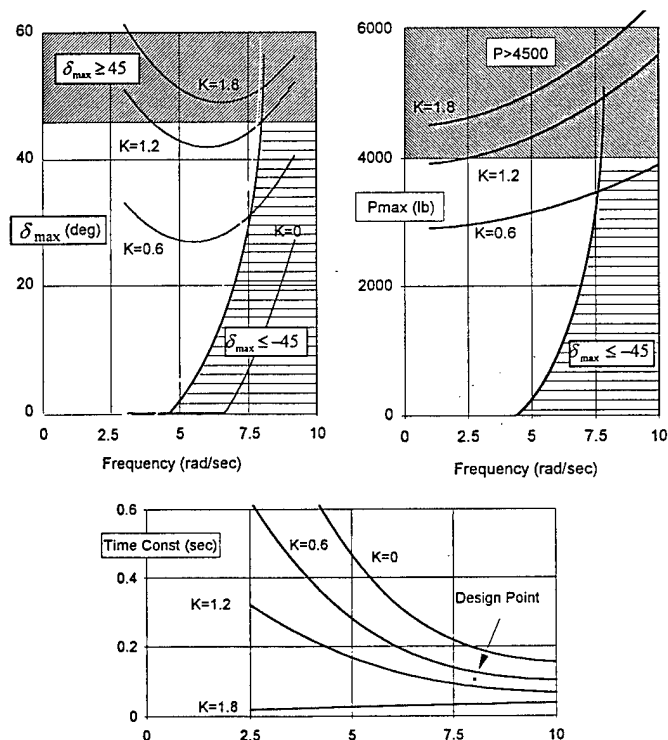

FIGURE 3. EFFECT OF LATERAL THRUST ON 4.2-G STEP RESPONSE 100,000 FT AT 4000 FT/SEC
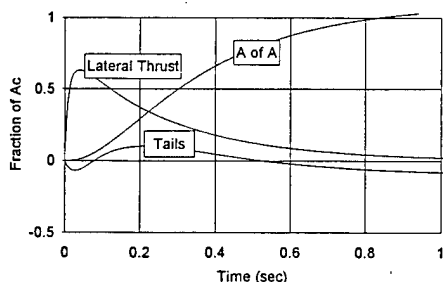
FIGURE 4. MANEUVER CONTRIBUTIONS DUE TO LATERAL THRUST AND AERODYNAMIC LIFT

FIGURE 5. UNDAMPED NATURAL FREQUENCY AND GAIN K

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Having established K and ω_O the values of k_1 and k_2 follow from Equations (17) and (18). Critical damping with $\xi=1$ is assumed throughout. Shown at the left in Figure 5 is the maximum positive tail deflection (leading edge up) occurring during a step command of 10.5 g's for a range of undamped natural frequencies and several values of K. At the right is the peak lateral thrust demanded and finally in the lower part of the figure is the time constant. Combinations of K and ω_O in the shaded zones result in the constraint violations noted. The minimum possible time constant of 0.11 sec therefore occurs for $\omega_O=8$ rad/sec and $K=0.9$. The peak thrust required is 4000 lb and the tails undergo excursions reaching 45 deg in both directions. With tails only, the minimum time constant is 0.45 sec and occurs for ω_O about 4.5 rad/sec (exceeding this frequency results in the peak negative tail excursion exceeding 45 deg). Similar calculations were performed for the three flight conditions previously noted, and the results are summarized in Table 2 and in Figure 6.

TABLE 2. AUTOPILOT PARAMETERS
AND TIME CONSTANT

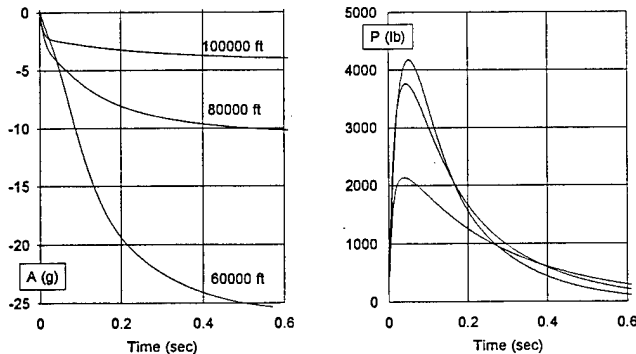| H Kft	100	80	60
A_C g's	4.2	10.5	26
K	1.5	0.9	0.25
ω_O rad/s	5.0	8.0	10.0
α_T deg	30	30	30
τ sec	0.1	0.11	0.16

FIGURE 6. STEP RESPONSES FOR THE THREE FLIGHT CONDITIONS

INFLUENCE OF THE LATERAL THRUST MOMENT

To this juncture we have held the location of the point of application of the lateral thrust at the constant value $b=2$ calibers ahead of the missile center of gravity. Here we give some indication of how close to the optimum value of b this location is. Attention is focused on the 10.5-g (80,000-ft) flight condition. The minimum time constant, or the smallest value of τ obtained without violating the previously specified constraints P (4000 lb) and δ(45 deg), is plotted against b in Figure 7.
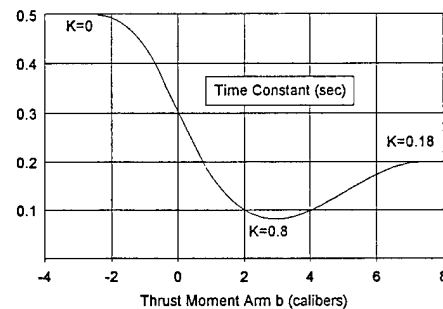

FIGURE 7. INFLUENCE OF THRUST MOMENT ON MINIMUM ACHIEVABLE TIME CONSTANT

The smallest minimum τ of about 0.1 occurs for b approximately 2 calibers, which is the nominal design case. When the thrust is applied 6 calibers forward of the center of gravity, the magnitude of P is limited by the ability of the tails to maintain stability and the minimum τ is 0.19. When P is applied over the center of gravity, the thrust provides no direct contribution to AOA build-up and the minimum τ is 0.30. Specific step response curves for several b's are shown in Figure 8. Note in particular the tails-only result in the upper part of this figure.
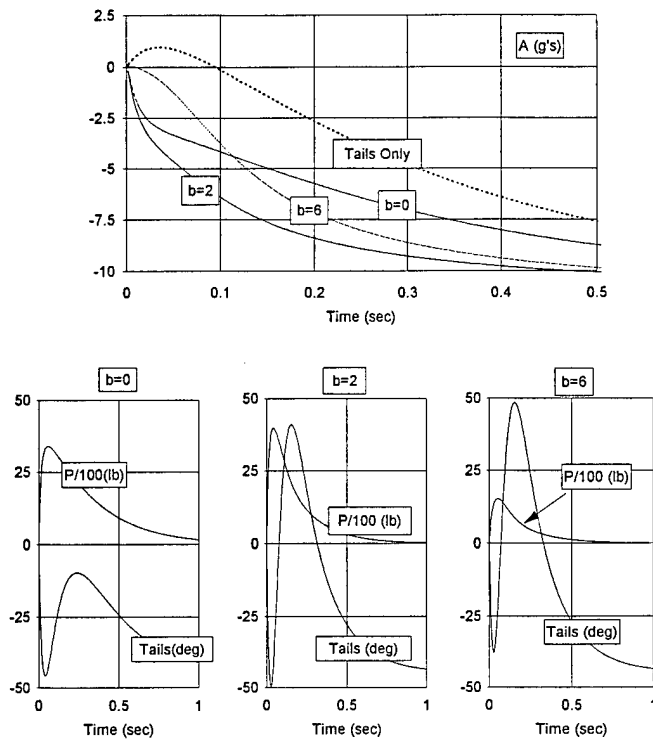
FIGURE 8. STEP RESPONSES FOR VARIOUS THRUST MOMENT ARMS

FIGURE 9. MISS AGAINST WEAVING TARGET, SINGLE-LAG THE

MISS DISTANCE PERFORMANCE

Most of the published work on proportional navigation guidance focuses on the miss due to (a) heading error, (b) step maneuver, (c) constant target evasion, and (d) constant target weave. The weaving target, even in the case where the weave frequency and amplitude are constant, presents the greatest challenge. The main reason for this is that the miss of the proportional navigation interceptor against the weaving target is always theoretically finite. In the other cases mentioned above, the miss becomes zero as the homing time and the maneuver capability of the interceptor become sufficiently large. This is not true against the weaving target. In this case when the missiles tracking head, noise filter, and autopilot dynamics are replaced by a single-lag time constant, the maximum possible miss with unlimited maneuver capability and infinite homing time was given previously as Equation 1 for a navigation constant of 3. In this equation ω_O is the weave frequency in rad/sec and A is the weave acceleration amplitude in ft/sec^2. This result is given in Figure 9 for a 10-g weave for three values of τ.
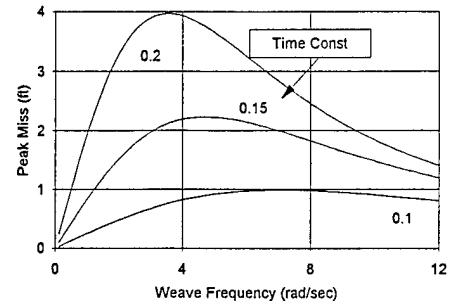
The miss is greatest when the target weave frequency is in the range 0.5 to 1 Hz. While the single lag theory is of immense value in predicting the relative importance of the three parameters that govern the noise-free miss, it should not be expected to yield accurate results for the non-exponential step response functions that characterize the present control scheme. However, from Figures 3 and 6, it may be seen that the 26-g blended-control step response at 60,000 ft does in fact approximate single-lag dynamics with τ=0.16. For this case the *3-DOF* miss calculations should agree quite well with the simple theory.

The miss distance study has been performed in the following manner. For a particular target (e.g. the 8-g amplitude weave at 0.5 Hz at 60,000 ft altitude) the peak miss is calculated over a range of initial phase angles of the weave and for a homing time of 3 sec which, in comparison with the interceptor time constant, is effectively infinite. Command saturation in this case is at the 26-g level corresponding to a trim *AOA* of 30 deg and the lateral thrust available is 4000 lb. The tails are limited to 45 deg in both directions. Transition from tail control to blended control occurs at Tg_0 and the results are plotted in Figure 10 for the three target cases. Against the 8-g target (interceptor saturation 26-g) the miss with tails only *(Tgo=0)* is 19 ft. With *Tgo* greater than 0.5, it is 3 ft in good agreement with the single-lag theory for the reasons mentioned. Against the 3-g target (saturation *10.5-g)* the miss with tails only is 12 ft. With *Tgo* greater than 0.5, it is zero. And finally against the 1-g target at 100,000 ft (saturation 4.2-g), the miss with tails only is 6 ft. With *Tgo* greater than 0.5, it is again zero.
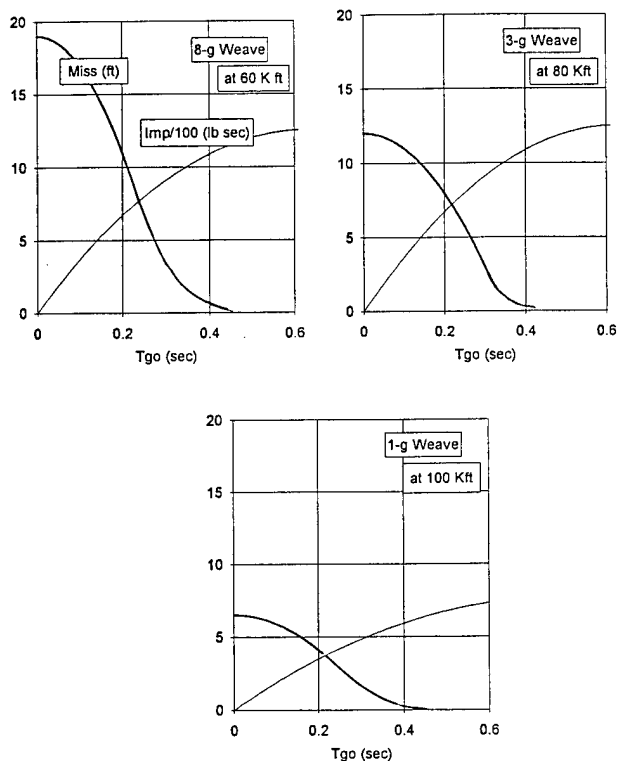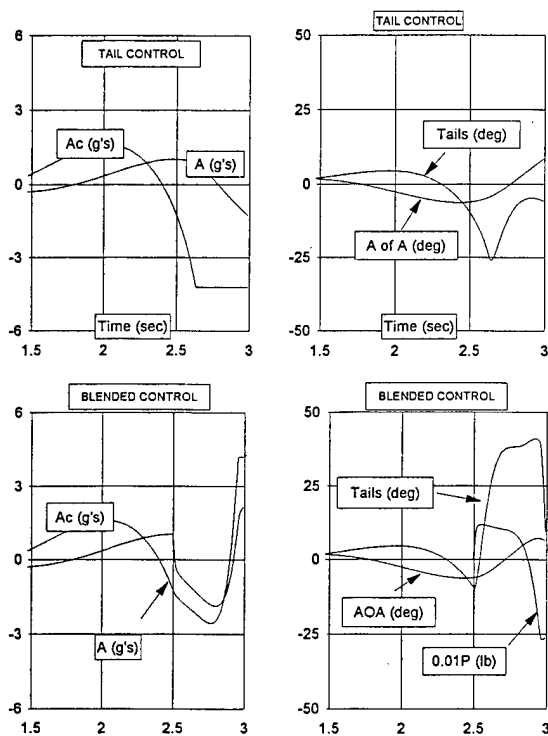
FIGURE 10. TAIL AND BLENDED CONTROL COMPARISON OF MISS

FIGURE 11 TAIL AND BLENDED CONTROL EXAMPLE OF 3-DOF CALCULATONS

It may thus be concluded that transition from tail control to blended control should occur with no less than 0.5 sec of flight time remaining to intercept. Noise-free miss against the postulated targets is then zero at 80,000 ft and above. It is 2 to 3 ft against the 8-g weave at 60,000 ft. The reason for this, of course, is that the lateral thrust, which is limited to 4000 lb, becomes less important compared to the aerodynamic maneuver forces as the peak maneuver demand increases. The total lateral thrust impulse for each target case is also shown in Figure 10. Figure 11 gives a detailed example of the 3 -DOF calculations, first for tail control all the way and second for blended control with *Tgo=0.5*. Achieved maneuver relative to command maneuver is dramatically improved with the new control concept.

CONCLUSIONS

TBM's may experience severe spiral maneuvers as they reenter the earth's atmosphere. It has been estimated that these maneuvers may increase in magnitude from 1 to 10-g's as the vehicle descends in altitude from 100 to 60 kft. Further, the circular frequency of the spiraling may be expected to approximate *0.5* Hz, which is within the worst range of evasive weave frequencies for the proportional navigation interceptor. To hit such targets, an interceptor time constant of about 0.1 sec is required. At the altitudes mentioned, this is impossible with aerodynamic control. The present study investigates the feasibility of a high-speed interceptor employing lateral thrust to augment aerodynamic responsiveness as a means of achieving the level of performance required. The proposed concept employs aerodynamic tails for most of the terminal flight. Transition to blended control using the tails in combination with lateral thrust occurs shortly before intercept. A simple theoretical basis for blended autopilot control is given.

The interceptor considered in the study weighs *850* lb and is 15 ft in length. A terminal speed of 4000 ft/sec is considered. It is shown that the maneuver performance required may be achieved by incorporating within the airframe, at a distance *b*=2 calibers ahead of the center of gravity, a proportional thrust device capable of developing a peak lateral thrust level of 4000 lb. Particular attention is given in the study to the importance of the thrust moment arm *b*. The time of transition to blended control should be as late in flight as possible. In

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this regard, *0.5* sec of blended autopilot control is suffi-
cient to achieve essentially zero noise-free proportional
navigation miss against the targets considered. Total
thrust impulse required is about 1200 lb-sec. The study is
preliminary and no account has been taken of jet interfer-
ence effects.

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